

Flavor versus mass eigenstates in neutrino asymmetries: implications for cosmology.

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We show that, if they exist, lepton number asymmetries (L_α) of neutrino flavors should be distinguished from the ones (L_i) of mass eigenstates, since Big Bang Nucleosynthesis (BBN) bounds on the flavor eigenstates cannot be directly applied to the mass eigenstates. Similarly, Cosmic Microwave Background (CMB) constraints on mass eigenstates do not directly constrain flavor asymmetries. Due to the difference of mass and flavor eigenstates, the cosmological constraint on the asymmetries of neutrino flavors can be much stronger than conventional expectation, but not uniquely determined unless at least the asymmetry of the heaviest neutrino is well constrained. Cosmological constraint on L_i for a specific case is presented as an illustration.

INTRODUCTION

A large lepton number asymmetry of neutrinos is an intriguing possibility with respect to its capability of resolving several non-trivial issues of cosmology (see for example [1–3]), but has been known to be constrained tightly by BigBang nucleosynthesis (BBN) [4, 5]. Interestingly, in a recent paper [6] it has been shown that, even if BBN constrains the lepton number asymmetry of the electron-neutrino very tightly such as $L_e \lesssim \mathcal{O}(10^{-3})$, much larger muon- and tau-neutrino asymmetries of $\mathcal{O}(0.1 - 1)$ are still allowed as long as the total lepton number asymmetry is sizeable. Such large asymmetries are expected to be constrained mainly by cosmic microwave background (CMB) via the extra neutrino species ΔN_{eff} [7].

If asymmetric neutrinos have a thermal distribution, their contribution to ΔN_{eff} is expressed as

$$\Delta N_{\text{eff}} = \frac{15}{7} \sum_{\alpha} \left(\frac{\xi_{\alpha}}{\pi} \right)^2 \left[2 + \left(\frac{\xi_{\alpha}}{\pi} \right)^2 \right] \quad (1)$$

where $\xi_{\alpha} \equiv \mu_{\alpha}/T$ is the neutrino degeneracy parameter. Conventionally, the summation in Eq. (1) has been done with neutrino flavors ($\nu_{e,\mu,\tau}$ in case of only three active neutrinos). An implicit assumption here is that the extra radiation energy coming from asymmetric neutrinos are solely from flavor-eigenstates. However, due to neutrino flavor oscillations [9–12], the equilibrium density matrix is not diagonal in the flavour basis (as one naively expects, flavor eigenstates not being asymptotic states of the Hamiltonian) and their description in terms of only diagonal components (a more or less hidden assumption when assuming thermal distribution for flavors) cannot capture all the contributions to the extra radiation energy density [8]. On the other hand, well after their decoupling from a thermal bath, free-streaming neutrinos should be described as incoherent mass-eigenstates only. Hence, the appropriate estimation of ΔN_{eff} should be done exclusively with neutrino mass-eigenstates instead of flavor-eigenstates in Eq. (1).

In this letter, we argue that the equilibrium lepton

number asymmetry matrix reached by the BBN epoch is diagonal in the mass eigenstate basis and related to the one in flavor eigenstate basis simply by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, and show that the lepton number asymmetries of the mass eigenstates are different from those of flavors. A numerical demonstration is provided. Also, we discuss implications of a lepton number asymmetry on cosmological data such as CMB+SN Ia.

LEPTON NUMBER ASYMMETRIES OF NEUTRINO FLAVOR VS. MASS EIGENSTATES

The lepton number asymmetries of neutrinos in flavor basis can be defined as a matrix such as

$$\mathbf{L}_f = \frac{\rho - \bar{\rho}}{n_{\gamma}} \quad (2)$$

where $\rho/\bar{\rho}$ and n_{γ} are the density matrices of neutrinos/anti-neutrinos and the photon number density. In the very early universe, it is natural to assume that neutrinos are in interaction eigenstates (i.e., flavor eigenstates), since their kinematic phases are very small and collisional interactions to thermal bath are large enough to block flavor oscillations. Hence, if it were generated at very high energy, \mathbf{L}_f is likely to be diagonal and to remain constant. While oscillations are blocked, individual flavor lepton numbers are conserved. However, due to the fact that neutrinos are not massless and mix (according to the values of the mixing parameters and mass differences measured by a variety of experiments [13]), as the temperature of the radiation dominated universe drops below around $T \sim 15$ MeV, flavor oscillations become active. \mathbf{L}_f starts evolving at this epoch, and settles down to an equilibrium state finally at $T \sim 2 - 5$ MeV before BBN starts [4, 14–18], depending on the neutrino mass hierarchy. Here, we consider the case of the normal mass hierarchy with zero CP-violating phase.

Once it reaches its final equilibrium value, \mathbf{L}_f becomes time-independent. The shape of \mathbf{L}_f at the final equilibrium is determined by various effects including vacuum

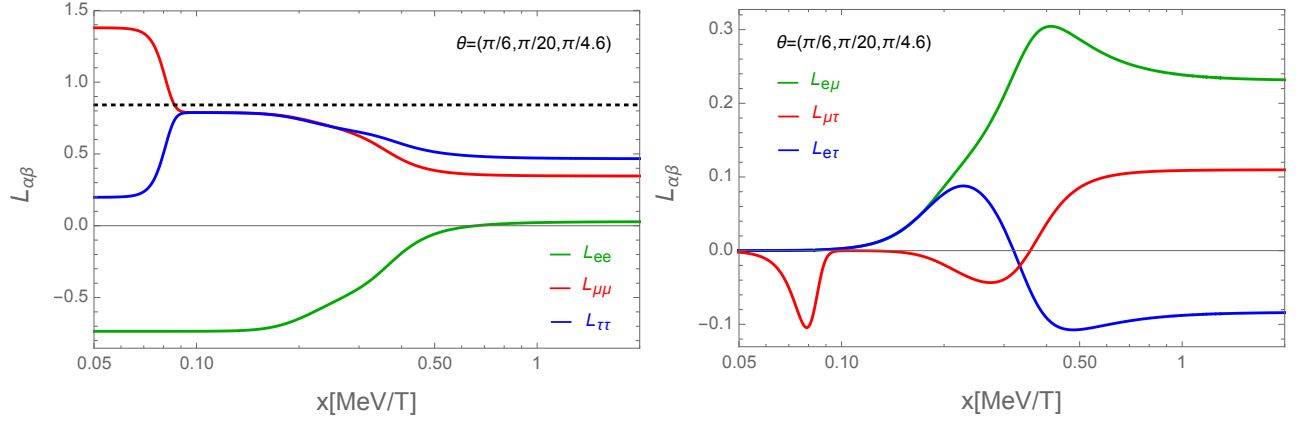


FIG. 1: Evolutions of \mathbf{L}_f for $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$ with θ_{ij} being the mixing angles in PMNS matrix, and $(\xi_e, \xi_\mu, \xi_\tau) = (-1.0, 1.6, 0.3)$. Left/Right: Diagonal/off-diagonal entries.

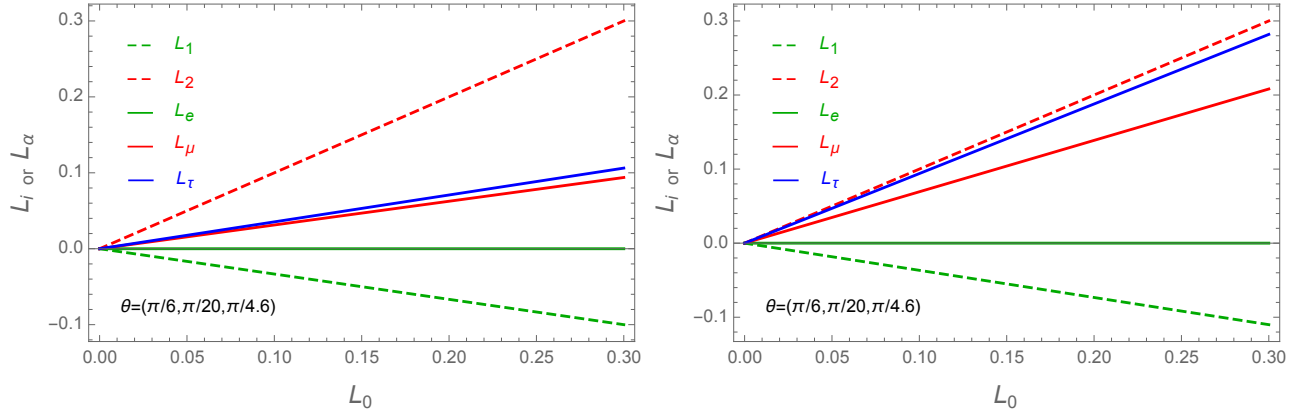


FIG. 2: Comparisons of lepton number asymmetries of both mass-eigenstates (L_i ; $i = 1, 2, 3$) and flavor-eigenstates (L_α ; $\alpha = e, \mu, \tau$) for $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$. Solid/dashed lines are the asymmetries of flavor/mass eigenstates. Left and right panels are showing two examples of \mathbf{L}_m leading to $L_e = 0$ satisfying BBN constraint. Left: $\mathbf{L}_m = \text{diag}(L_1, L_2, L_3) = (-t_1^2 L_0, L_0, 0)$. Right: $\mathbf{L}_m = \text{diag}(-(t_1^2 + t_1^2/c_1^2) L_0, L_0, L_0)$.

oscillations, MSW-like effect coming from charged lepton backgrounds, neutrino self-interactions, and collisional damping. So, it is difficult to predict analytically, and in practice is only accessible via numerical methods. However, all these effects except vacuum oscillations are active during particular windows in temperature and eventually disappear. Hence, the final shape of \mathbf{L}_f should be determined by vacuum oscillation parameters only. Note that the flavor states mixed by vacuum oscillation parameters are nothing but mass-eigenstates in flavor-basis. Therefore, the statistical equilibrium state of \mathbf{L}_f should be that of mass-eigenstates expressed in the flavor-basis.

Since in vacuum mass- and flavor-eigenstates are related to each other by the PMNS matrix, U_{PMNS} [19, 20], our argument implies that for a diagonalization matrix D , the matrix \mathbf{L}_m of asymmetries in mass basis is given

by

$$\mathbf{L}_m = D^{-1} \mathbf{L}_f D = U_{\text{PMNS}}^{-1} \mathbf{L}_f U_{\text{PMNS}} \quad (3)$$

implying

$$D = U_{\text{PMNS}} \quad (4)$$

On general grounds, at late times we do not expect \mathbf{L}_f to be diagonal. The operator responsible for the evolution of the density matrix is not diagonal, so that a diagonal density matrix will not be the asymptotic solution of those equations unless it is proportional to the identity matrix. Hence, generically the asymmetries of neutrino mass eigenstates differ from those of flavor, and this fact should be taken into account when observational constraints on lepton number asymmetries are considered.

In order to verify our argument, we solved numerically the quantum kinetic equations of neutrino/anti-neutrino

density matrices [21, 22] in a simplified way as done in Ref. [6]. An example is shown in Fig. 1 where one finds the evolutions of $L_{\alpha\beta}$, the (real) entries of \mathbf{L}_f . As shown in the right panel of the figure, the off-diagonal entries of \mathbf{L}_f do not disappear, making \mathbf{L}_m be different from \mathbf{L}_f . Also, we found that the numerical simulation reproduces the relation Eq. (4) quite precisely within errors of $\mathcal{O}(0.1)\%$ even at $x = 1$.

The differences between diagonal entries of \mathbf{L}_f and \mathbf{L}_m can be seen by expressing the former in terms of the latter. At first, L_e is given by

$$L_e = c_{13}^2 (c_{12}^2 L_1 + s_{12}^2 L_2) + s_{13}^2 L_3, \quad (5)$$

where $c_{ij}/s_{ij}/t_{ij} = \cos\theta_{ij}/\sin\theta_{ij}/\tan\theta_{ij}$ with θ_{ij} being the mixing angle in PMNS matrix. Since BBN requires $L_e \lesssim \mathcal{O}(10^{-3})$, we may set $L_e = 0$ for an illustration. In this case, L_μ and L_τ are given by

$$L_\mu = c_{23} [(1 - t_{12}^2)c_{23} - 2s_{13}s_{23}t_{12}] L_2 + [(1 - t_{13}^2)s_{23}^2 - t_{12}t_{13}^2c_{23}(2s_{13}s_{23} + t_{12}c_{23})] L_3, \quad (6)$$

$$L_\tau = s_{23} [(1 - t_{12}^2)s_{23} + 2s_{13}c_{23}t_{12}] L_2 + [(1 - t_{13}^2)c_{23}^2 + t_{12}t_{13}^2s_{23}(2s_{13}c_{23} - t_{12}s_{23})] L_3. \quad (7)$$

From Eqs. (6) and (7) with measured values of mixing angles [13], we find that $L_\mu \sim L_\tau$ for $|L_3| \lesssim |L_2|$, as shown in Fig. 2. One may think that it is also possible to have $|L_{\mu,\tau}| \ll |L_{2,3}|$ if $L_2 \sim -L_3$. However, our numerical testing showed that generically $\mathbf{Max}\{|L_{\alpha\beta}|_{\alpha \neq \beta}\} \lesssim \mathbf{Max}\{|L_{\alpha\alpha}|\}$. Hence, on general grounds one expects to have

$$\mathcal{O}(0.1) \lesssim \mathbf{Max}\{|L_i|\}/\mathbf{Max}\{|L_{\alpha\alpha}|\} \lesssim \mathcal{O}(1), \quad (8)$$

showing that it is critical to know at least two of L_i s in order to constrain L_μ and L_τ .

COSMOLOGICAL CONSTRAINTS

A large lepton number asymmetry in one or more neutrino species creates an extra radiation density in the universe relative to the standard contributions of photons and CP-symmetric active neutrinos, a form of so-called “dark radiation”. Extra relativistic degrees of freedom in cosmology have attracted considerable recent attention as a way to resolve the apparent discrepancy in measurement of the Hubble parameter from CMB data and type-Ia supernovae [7, 25–32]. In this section, we investigate the possibility that a primordial lepton asymmetry may provide a dark radiation density which can reconcile CMB and SNIa values for the Hubble parameter.

We consider two basic cases. The first is an eight-parameter Λ CDM+ ξ cosmology without contribution from primordial tensor fluctuations, with parameters

- Baryon density $\Omega_b h^2$.

- Dark matter density $\Omega_C h^2$.
- Angular scale of acoustic horizon θ .
- Reionization optical depth τ .
- Hubble parameter H_0 .
- Power spectrum normalization A_s .
- Scalar spectral index n_s .
- Lepton asymmetry ξ .

In the second case, motivated by models of early-universe inflation, we include the tensor/scalar ratio r as a ninth parameter to the fit. We assume a normal mass hierarchy for neutrinos, with one massive neutrino with mass $m_\nu = 0.06$ eV. Since the BBN constraint on L_e should be satisfied, we are not free to choose $|L_i| \gg |L_e|$ in an arbitrary way, but constrained to satisfy approximately

$$c_{12}^2 L_1 + s_{12}^2 L_2 + t_{13}^2 L_3 = 0, \quad (9)$$

coming from $L_e = 0$. As the simplest possibility, we may set $L_3 = 0$ leading to $L_1 = -t_{12}^2 L_2$. Then, for thermal distributions of two light mass eigenstates,

$$\begin{aligned} \Delta N_{\text{eff}} &= \frac{15}{7} \sum_{i=1,2} \left(\frac{\xi_i}{\pi} \right)^2 \left[2 + \left(\frac{\xi_i}{\pi} \right)^2 \right] \\ &\approx \frac{15}{7} \left(\frac{\xi_2}{\pi} \right)^2 \times \\ &\quad \left\{ (1 + t_{12}^4) 2 + [1 + (4 + t_{12}^4)t_{12}^4] \left(\frac{\xi_2}{\pi} \right)^2 \right\}, \quad (10) \end{aligned}$$

where ξ_i s are degeneracy parameters of each mass eigenstate, and $|\xi_i| \lesssim 1$ and $t_{12}^2 \ll 1$ were assumed. Strictly speaking, the late-time free-streaming neutrino mass-eigenstates are not in thermal distribution since they are linear combinations of thermal distributions of flavor-eigenstates. Hence, ξ_i s in Eq. (10) should be understood as effective degeneracy parameters. The error in ΔN_{eff} depends on the initial configuration of the lepton number asymmetries in flavor-basis, but it is expected to be of or small than $\mathcal{O}(10)\%$ for $|\xi_i| \lesssim 1$. We constrain the parameter space with: (a) Planck 2015 TT/TE/EE+lowTEB temperature and polarization data [7, 33], and the Bicep/Keck 2014 combined polarization data [34], and (b) CMB data combined with the Riess, *et al.* supernova data [27]. The allowed contours are calculated numerically using a Markov Chain Monte Carlo method with the cosmomc software package [35].¹ Curvature Ω_k is

¹ The data sets themselves contain multiple internal parameters, which we do not list here.

set to zero, and the Dark Energy equation of state is fixed at $w = -1$. For these constraints, we run 8 parallel chains with Metropolis-Hastings sampling, and use a convergence criterion of the Gelman and Ruben R parameter of $R - 1 < 0.05$.

Case 1: Λ CDM+ ξ

Figure 3 shows constraints on H_0 and ξ for the case of the eight-parameter Λ CDM+ ξ fit. We plot constraints from Planck+BICEP/Keck only (filled contours), and Planck+BICEP/Keck+Riess *et al.* (dashed contours). The CMB data alone show no evidence for nonzero neutrino chemical potential, with a 95%-confidence upper bound of $|\xi| < 0.78$, with $H_0 = 67.76 \pm 0.96$. For combined CMB and supernova data, there is weak evidence for a nonzero chemical potential, with $|\xi| = 0.61 \pm 0.28$ at 68% confidence, with $H_0 = 69.25 \pm 1.18$. The combined CMB+supernova data, however, should be interpreted with caution: as the filled contours illustrate, the CMB data and supernova data taken separately are barely compatible, with only a small overlap in the 95% confidence regions, even when dark radiation from a neutrino asymmetry is included as a parameter. Combining two fundamentally incompatible data sets in a Bayesian analysis is likely to give a biased fit, which is reflected in the best-fit values for the two cases, with the best-fit to CMB alone having $-\ln(\mathcal{L}) = 6794.59$, while the best-fit for the the combined CMB+supernova data is measurably worse, with $-\ln(\mathcal{L}) = 6798.09$. For the CMB data alone, including lepton asymmetry, the 95%-confidence upper bound on the Hubble parameter is $H_0 < 69.8$. This can be compared with a 95%-confidence *lower* bound from Type-Ia supernovae of $H_0 > 69.8$. We therefore conclude, contrary to existing claims in the literature [23–26], that inclusion of dark radiation does not provide a consistent mechanism for reconciling the discrepancy between CMB and supernova data. Furthermore, there is no evidence for a nonzero lepton asymmetry from current data.

Case 2: Λ CDM+ ξ + r : constraints on inflation

Figure 4 shows parameter constraints on the nine-parameter case, with tensor perturbations included, consistent with generic expectations from inflation. Constraints in the H_0 , ξ parameter space are extremely similar to the case of no tensors, which is reasonable considering the upper bound of $r < 0.07$ obtained from Planck+BICEP/Keck data [36]. In this case we obtain a 95%-confidence upper bound on the lepton asymmetry of $|\xi| < 0.78$, and $|\xi| = 0.59 \pm 0.30$ for Planck+BICEP/Keck+SN Ia at 68%-confidence. The best-fit to CMB alone is $-\ln(\mathcal{L}) = 6794.53$, and CMB+SN Ia is $-\ln(\mathcal{L}) = 6798.27$, nearly identical to the

no-tensor case. As in the no-tensor case, we conclude that here is no evidence for dark radiation from a lepton asymmetry. Constraints on inflationary potentials are shown in the right-hand panel of Fig. 4, which can be compared to Fig. 1 of Tram, *et al.* [32]. Our constraints here are considerably tighter. The difference is that here we include the BICEP/Keck polarization data, which results in a considerably stronger constraint on the parameter space than that provided by Planck alone. Of particular note, our constraint rules out power-law inflation, with $V(\phi) \propto e^{\phi/\mu}$, even in the presence of dark radiation, which is allowed by the constraints of Tram, *et al.* Ref. [37] reaches a similar conclusion based on constraints from Planck on σ_8 and the reionization optical depth τ_{reio} .

CONCLUSIONS

In this letter, we argued that, when lepton number asymmetries of neutrinos in flavor basis are mixed among themselves due to neutrino oscillation in the early universe before BBN, the asymmetries at the final equilibrium are well described in the basis of mass eigenstates, which are related to flavor eigenstates by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. That is, the matrices of lepton number asymmetries in mass- and flavor-basis (\mathbf{L}_m and \mathbf{L}_f , respectively) are related as

$$\mathbf{L}_m = U_{\text{PMNS}}^{-1} \mathbf{L}_f U_{\text{PMNS}}, \quad (11)$$

where U_{PMNS} is the PMNS matrix, and \mathbf{L}_m appears to be diagonal. We demonstrated this argument by a numerical simulation, and showed analytically that the asymmetries of mass-eigenstates can be even larger than those of flavor-eigenstates.

Conventionally, the constraint on the lepton number asymmetries of neutrino flavors has been associated with neutrino flavor-eigenstates, counting their contributions to the extra radiation energy density ΔN_{eff} . However, our argument and finding showed that a proper estimation should be done with neutrino mass-eigenstates in order not to miss the contributions of flavor-mixed states in flavor-basis, and the resulting ΔN_{eff} can be larger than the one estimated with flavor-eigenstates only.

As shown in Ref. [6] and in this work, in principle ΔN_{eff} can be of $\mathcal{O}(0.1 - 1)$ just from asymmetric neutrinos without resorting to an unknown “dark radiation”. Such a large ΔN_{eff} has been considered in literature as a possible solution to the discrepancy of the measured expansion rate H_0 in CMB and SN Ia data. In analyses of cosmological data, typically, if ΔN_{eff} is from asymmetric neutrinos, the neutrino degeneracy parameters have been taken in an arbitrary way without distinguishing mass- and flavor-eigenstates, although implicitly the lepton number asymmetry (L_e) of electron-neutrinos must be assumed to be small to satisfy BBN constraint. We

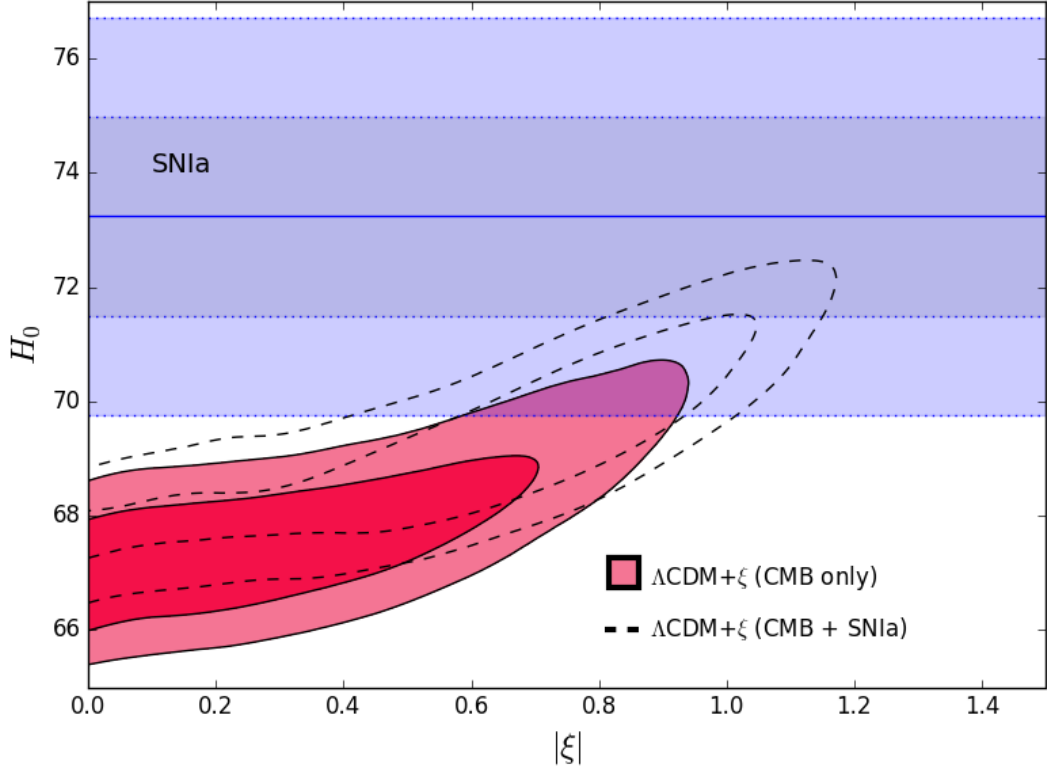


FIG. 3: Constraints on H_0 and ξ for the eight-parameter $\Lambda\text{CDM}+\xi$ case. Filled contours show the 68% (dark red) and 95% (light red) constraints from Planck+BICEP/Keck alone. Dashed contours show the corresponding constraints with the addition of the Riess *et al.* supernova data. The constraint on H_0 from the supernova data alone, $H_0 = 73.24 \pm 1.74$ [27] is shown by the filled regions, with 1σ limits in lavender, and 2σ limits in grey.

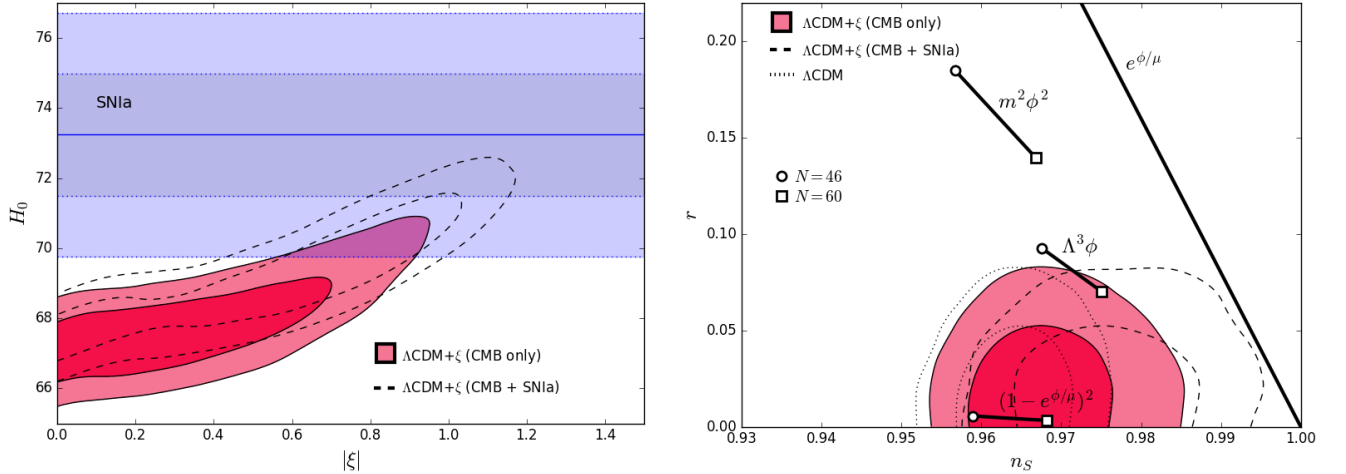


FIG. 4: CMB constraints on lepton number asymmetries for the nine-parameter model including tensor perturbations. Contours are 68% and 95% uncertainties from CMB-only (red-shaded regions), and CMB+supernovae (dashed lines). *Left*: Constraint as a function of ξ and H_0 . The filled region is the Riess *et al.* constraint on H_0 from supernovae. *Right*: Constraint on the spectral index n_s and tensor-to-scalar ratio r , plotted with the predictions of representative choices of inflationary scalar-field potential. Dotted contours for $\Lambda\text{CDM}+r$, with fixed $\xi = 0$. This can be compared to Fig. 1 of Tram, *et al.* [32].

showed that this approach is inconsistent unless the lepton number asymmetries (L_i) of mass-eigenstates which

are relevant for CMB data for example are constrained to satisfy

$$L_e = c_{12}^2 L_1 + s_{12}^2 L_2 + t_{13}^2 L_3 \approx 0, \quad (12)$$

for $|L_e| \lll |L_i|$. Also, analyzing cosmological data (CMB only or CMB+SN Ia), we found that CMB data alone show no evidence for nonzero neutrino lepton number asymmetries, with 95% CL upper bound of $|\xi| \leq 0.78$ at 95% CL as the degeneracy parameter of the dominant mass eigenstate. For combined CMB and SN Ia data, there is weak evidence for nonzero lepton number asymmetries, with $|\xi| = 0.61 \pm 0.28$ at 68% CL, but the fit becomes worse relative to the case of CMB data alone. So, even if large lepton number asymmetries may fit the data, it does not look preferred.

As the final remark, because of the degeneracy of \mathbf{L}_m for a given ΔN_{eff} , the bound on ΔN_{eff} can not be uniquely interpreted in terms of the asymmetries of neutrino flavors (specifically L_μ and L_τ of muon- and tau-neutrinos), unless the impact on small scale power spectrum is sensitive enough to distinguish at least the contribution of the heaviest neutrinos.

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